STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

CAL C: FUNCTIONS & GRADIENT

Functions of Several Variables:

- Just as a typical one-dimensional function maps an input (x, horizontal axis) to an output (y, vertical axis), so a two-dimensional function maps inputs (x *and* y) to an output (z, height). All rules still apply to higher-order functions and many properties scale.

Directional Derivatives and Gradient:

- Directional derivatives describe how rapidly a function is changing in a certain direction as defined by the unit vector of $\boldsymbol{v}$. A typical partial derivative is a specific directional derivative, with the unit vector in the $\boldsymbol{i}$ or $\boldsymbol{j}$ direction. If you are given a vector **u** that is not a unit vector, then the unit vector **v** would be **u**/[**u**].
- Gradients define the rate of change in the direction that water would flow along the surface; that is, the direction of greatest increase/decrease at a specific point. A vector orthogonal to the gradient experiences no change. This is analogous to a contour map.

The gradient of f is noted by the Greek letter "del":

$$\nabla f = \frac{\partial f}{\partial x}\boldsymbol{i} + \frac{\partial f}{\partial y}\boldsymbol{j}$$

The directional derivative at point P_0 in the direction of $\boldsymbol{u} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$:

$$\left(\frac{\partial f}{\partial s}\right)\bigg|_{\boldsymbol{u},P_0} = (\nabla f)|_{P_0} \cdot \boldsymbol{u}$$

Extreme Values:

- First, determine the location (a, b) such that $f_x = f_y = 0$. *Note: this often requires factoring.*
- Next, apply the second derivative test to determine which feature is located at (a, b).
- If the discriminant of f is < 0 (negative), f has a saddle point at (a, b).
- If the discriminant of f is $= 0$, the test is inconclusive.
- If the discriminant of f is > 0 (positive), evaluate the following rules:
    - If $f_{xx} < 0$ (negative) then there is a local maximum at (a, b).
    - If $f_{xx} > 0$ (positive) then there is a local minimum at (a, b).
- The discriminant is defined as $f_{xx} \cdot f_{yy} - f_{xy}^2$, which can be remembered using $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$

For more information, visit a tutor. All appointments are available in-person at the Student Success Center, located in the Library, or online. Adapted from Hass, J., Weir, M.D., & Thomas, G.B. (2012). *University Calculus: Early Transcendentals* (2nd ed.). Boston: Pearson Education.